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Correction on the paper Triple positive solutions of n-th order impulsive integro-differential equations

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Abstract

This addendum concerns the paper of the above title found in EJQTDE No. 57 (2011). The example in Section 4 was not correct. The following example is a correction given by the authors. We regret any inconvenience which this may have caused any reader.

1 Correction

The example in Section 4 of the original text, i.e. problem (13), is not written correctly. The following example is a correction given by the authors.

Consider the second order impulsive integro-differential equation

$$\begin{cases} u''(t) = f(t, u(t), u'(t), (Tu)(t), (Su)(t)), \ \ \forall t \in J, \ t \neq 2^k \ (k = 0, 1, 2, \cdots); \\ \Delta u|_{t=2^k} = 2^{-k}[u(2^k)]^2(15 + [u(2^k) + u'(2^k)]^2)^{-1}, \ \ (k = 0, 1, 2, \cdots), \\ \Delta u'|_{t=2^k} = 4^{-k}[u'(2^k)]^{3/2}(5 + (u(2^k) + u'(2^k))^{3/2})^{-1}, \ \ (k = 0, 1, 2, \cdots), \\ u(0) = 0, \ u'(\infty) = 2u'(0). \end{cases} \tag{1}$$

Here Tu and Su are given by

$$\begin{aligned} (Tu)(t) &= \int_0^t e^{-(t+1)s} u(s)ds = \int_0^t K(t, s)u(s)ds; \\ (Su)(t) &= \int_0^\infty e^{-2s} \sin^2(t - s)u(s)ds = \int_0^t H(t, s)u(s)ds \end{aligned}$$

with $K(t, s) = e^{-(t+1)s}$, $H(t, s) = e^{-2s} \sin^2(t - s)$, and, with $U = (u_0, u_1, u_2, u_3)$, f is the function

$$f(t, U) = \begin{cases} 18e^{-2t}e^{-2(10-u_0)(10-u_1)}g(U), & U \in [0, 10) \times [0, 10) \times [0, \infty) \times [0, \infty), \\ 18e^{-2t}g(U), & \text{otherwise}. \end{cases}$$

with $g(U) = g(u_0, u_1, u_2, u_3) := \left(\frac{1+3u_0+4u_1+5u_2+6u_3}{2+u_0+u_1+u_2+u_3}\right)^2, \forall t \in J = [0, \infty), u_i \geq 0$ $(i = 0, 1, 2, 3)$. It is clear that g is a continuous positive function and

$$g(t, u(t), u'(t), (Tu)(t), (Su)(t)) = \left(\frac{1 + 3u(t) + 4u'(t) + 5(Tu)(t) + 6(Su)(t)}{2 + u(t) + u'(t) + (Tu)(t) + (Su)(t)}\right)^2.$$

Conclusion. The problem (1) has at least three positive solutions $x_1(t), x_2(t), x_3(t)$ such that

$$\begin{aligned}
&\|x_j\|_D \leq 2160 \quad \text{for } j = 1, 2, 3;\\
&10 < \min\left\{\min_{t\in[\frac{1}{2},\infty)} x_1^{(i)}(t) : i = 0, 1\right\};\\
&8 < \max\left\{\sup_{t\in[0,1]} x_2^{(i)}(t) : i = 0, 1\right\} \text{ with } \min\left\{\min_{t\in[\frac{1}{2},\infty)} x_2^{(i)}(t) : i = 0, 1\right\} < 10;\\
&\max\left\{\sup_{t\in[0,1]} x_3^{(i)}(t) : i = 0, 1\right\} < 8.
\end{aligned}$$

Proof. Let $E = DPC^{n-1}[J, \mathbb{R}]$, $P = DPC^{n-1}[J, \mathbb{R}_+]$. Thus, (1) can be regarded as BVP of the form (1) of the original text in E. In this case, $t_{k+1} = 2^k$ $(k = 0, 1, 2, \cdots)$, $\rho = 2$, in which

$$\begin{aligned}
&I_{0k}(u_0, u_1) = 2^{-k}u_0^2(15 + (u_0 + u_1)^2)^{-1},\\
&I_{1k}(u_0, u_1) = 4^{-k}u_1^{3/2}(5 + (u_0 + u_1)^{3/2})^{-1}, \quad \forall u_0 \geq 0, u_1 \geq 0, (k = 0, 1, 2, \cdots).
\end{aligned}$$

Obviously, $I_{0k}, I_{1k} \in C[J, \mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+]$ $f \in C[J \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+]$. Moreover,

$$\int_0^t e^{-(t+1)s}ds = -\frac{e^{-(t+1)t}}{t+1} + \frac{1}{t+1} < 1, \quad \int_0^\infty e^{-2s}\sin^2(t-s)ds \leq \frac{1}{2}.$$

Since $e^{-t}\int_0^t e^{-(t+1)s}e^s ds \leq te^{-t}$, $e^{-t}\int_0^t e^{-2s}\sin^2(t-s)e^s ds \leq e^{-t}$, $\forall t \in J$, we have

$$\begin{aligned}
k^* &= \sup_{t\in J}\left(e^{-t}\int_0^t e^{-(t+1)s}e^s ds\right) \leq \sup_{t\in J}(te^{-t}) = \frac{e^{-1}}{2},\\
h^* &= \left(e^{-t}\int_0^\infty e^{-2s}\sin^2(t-s)e^s ds\right) \leq \sup_{t\in J}(e^{-t}) = 1.
\end{aligned}$$

Hence, condition (H1) is satisfied. From the definitions of f, I_{0k} and I_{1k} we have

$$0 \leq f(t, u_0, u_1, u_2, u_3) \leq 648e^{-2t}\left(\frac{1 + u_0 + u_1 + u_2 + u_3}{2 + u_0 + u_1 + u_2 + u_3}\right)^2 < 648e^{-2t}$$

for any $t \in J$, $u_i \geq 0$ $(i = 0, 1, 2, 3)$.

$$0 \leq I_{0k}(u_0, u_1) \leq 2^{-k} \frac{(u_0 + u_1)^2}{15 + (u_0 + u_1)^2} \leq 2^{-k},$$
$$0 \leq I_{1k}(u_0, u_1) \leq 4^{-k} \frac{(u_0 + u_1)^{3/2}}{5 + (u_0 + u_1)^{3/2}} \leq 4^{-k}$$

for any $u_0 \geq 0, u_1 \geq 0$ $(k = 0, 1, 2, \cdots)$.

We now take $\rho = 2, \lambda(t) = c(t) = e^{-2t}, \eta_{0k} = \mu_{0k} = 2^{-k}, \eta_{1k} = \mu_{1k} = 4^{-k}$, then $\lambda^* = c^* = \frac{1}{2}, \eta_0^* = \mu_0^* = 1, \eta_1^* = \mu_1^* = \frac{1}{3}, L = \frac{10}{3}$. Take $a = 8, b = 10, d = 648$, then the condition (H2) holds.

Take $l = \frac{1}{2}$, then $k_1 = 1, k_2 = \frac{1}{2}$. Take $m = 3$. Since $t_1 = 1$, $\lambda_0 = e^{-2}$. For $0 \leq t \leq \frac{1}{2}$ and $u_0 \geq 10, u_1 \geq 10, u_2 \geq 0, u_3 \geq 0$, since the function $\alpha(t) = \frac{3^{-1}+t}{2+t}$ for $t \geq 0$ is increasing, we have

$$\begin{aligned} f(t, u_0, u_1, u_2, u_3) &\geq 18e^{-2t} \times 9 \left(\frac{3^{-1} + u_0 + u_1 + u_2 + u_3}{2 + u_0 + u_2 + u_2 + u_3} \right)^2 \\ &\geq 162e^{-1} \left(\frac{20 + 3^{-1}}{22} \right)^2 > 20 = \frac{k_1 b}{l}. \end{aligned}$$

This implies that the condition (H3) is true.

Take $q_0 = 1$, then $\delta = \frac{3}{10e}$. if $0 \leq u_0 \leq 8, 0 \leq u_1 \leq 8$, then $0 \leq u_2 \leq 8, 0 \leq u_3 \leq 4$. From this and the fact that the function $\frac{t}{t+1}$ is increasing it follows that

$$\frac{1 + 3u_0 + 4u_1 + 5u_2 + 6u_3}{2 + u_0 + u_1 + u_2 + u_3} \leq \frac{6(1 + u_0 + u_1 + u_2 + u_3)}{2 + u_0 + u_1 + u_2 + u_3} \leq \frac{29}{5} = 5.8.$$

Thus, we get

$$\begin{aligned} f(t, u_0, u_1, u_2, u_3) &= 18e^{-2t} e^{-2(10-u_0)(10-u_1)} \left(\frac{1 + 3u_0 + 4u_1 + 5u_2 + 6u_3}{2 + u_0 + u_1 + u_2 + u_3} \right)^2 \\ &\leq 18e^{-2t-8} (5.8)^2 < \frac{24}{10e} e^{-2t} = a\delta c(t). \\ I_{0k}(u_0, u_1) &= 2^{-k} \frac{u_0^2}{15 + (u_0 + u_1)^2} \leq \frac{64}{79} \times 2^{-k} < a\delta\mu_{0k}, \\ I_{1k}(u_0, u_1) &= 4^{-k} \frac{u_1^{3/2}}{5 + (u_0 + u_1)^{3/2}} \leq \frac{8^{3/2}}{5 + 8^{3/2}} \times 4^{-k} < a\delta\mu_{1k}. \end{aligned}$$

So, condition (H4) is satisfied. Consequently, our conclusion follows from Theorem 1 since f is a positive function so x_3 is not the zero solution.

2 Acknowledgment

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